Code: MAT/05
Credits: 9
Matter: Calculus I
Main language of instruction: Italian
Other language of instruction: English

Teaching Staff

Head instructor
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Introduction

1. *Objective of the course :*

Calculus I is one of the modules of the first year of the three-year degrees in Civil Engineering, Electronic and IT Engineering and Industrial Engineering. The syllabus consists of four parts: real numbers and functions, limits and continuous functions, differential calculus, integrals. The first section is dedicated to the study of the real numbers and their basic topological features, together with a review of functions and their graphs, fundamental for the remainder of the module. The concept of limit is introduced in the second section. Continuity of a function is also defined, followed by the introduction of techniques to compute limits of functions and the study of properties of continuous functions. The definition of the derivative of a function is given in the third section together with two main applications: the study of a curve and the calculation of limits. The final section is dedicated to the study of Riemann integral and of techniques to integrate several families of functions.

Objectives

2. *Course Structure:*

The aim of the course is to acquire notions and be able to analyse real functions of a real variable. In particular, the main objective of the course is to be able to perform a qualitative study of the graphs of said functions and to compute related areas through integrals.

Competencies:

A. Knowledge and understanding.
Topics studied in this course include real numbers, topology of R, limits, differential and integral calculus of real functions of a real variable. The student acquires the skill of performing a rigorous logical reasoning, and the ability to use specific language and techniques.

B. Applying knowledge and understanding.
The student will be able to solve moderate to difficult problems, and to complete proofs that are only sketched and have been discussed in lectures. In particular, the student will be able to use methods and reasoning proper of the discipline to solve problems such as determining the graph or computing antiderivatives of a function. Moreover, the student will be able to recognise if and when a theorem is applicable in specific situations.

C. Making judgements.
The student will be able to recognise correct proofs and identify fallacies in logical arguments. In particular, the student will be able to determine the best strategies to solve some problems that are typical of the discipline.

D. Communication skills.
The student will gain the ability to discuss the notions, related problems, ideas and techniques acquired within this course both with a specialist and with a non-specialist audience, utilising a clear and rigorous language.

E. Learning skills.
At the end of the course, the student will be able to understand the interactions between the methods analysed and the mathematical models discussed in other parallel or forthcoming courses. In particular, the student will be able to use the knowledge from this course and apply it independently in subsequent courses in mathematical analysis.

Syllabus

3. *Programme of the course:*

Subject 1. Real Numbers and functions.
The real line, its ordering, absolute value and distance. Special subsets of the real line: intervals, number sets and their properties (Archimedean property of the natural numbers, density of the rational numbers, and sequences. Boundedness, upper and lower bounds, maxima and minima,

supremum and infimum. Completeness of the reals. Accumulation points, open and closed sets, closure, limit points and Bolzano-Weierstrass theorem. Functions and real functions, domain, restriction and extension. Graphs of functions, special points and intervals. Piecewise defined functions: domain and graph.

Subject 2. Limits and continuous functions.
Informal idea of limit, images of sequences through functions. Formal definition of a limit through sequences. Uniqueness of the limit, right and left limits, limits of monotonic functions, sandwich theorem for limits of functions and its application. Mention of alternate definitions of limit. Sign permanence theorem and its applications. Continuity of a function at a point and in an interval. Intermediate value theorem and existence of zeros, examples and counterexamples and applications. Continuity of the inverse function, examples and counterexamples. Weierstrass theorem, examples and counterexamples. Limits: elementary limits, compositions and substitution, limits of algebraic operations and indeterminate forms. Limits of rational functions and generalisations. Asymptotic equivalence and o-notation. Study of continuity and continuous extensions, with an emphasis on piecewise defined functions.

Subject 3. Differential calculus.
Derivatives: definition and geometric meaning. Techniques of differentiation. Derivatives of trigonometric functions. Derivatives of exponential functions. Composite functions and the chain rule. Inverse functions and their derivatives. The Mean Value Theorem. L'Hopital's Rule. Taylor polynomials. Graphing functions: first derivative test; concavity and second derivative test; infinite limits and asymptotes; optimization.

Subject 4. Integrals.
Antiderivatives. Area as limit of a sum. Riemann Sums and the definite integral. The Fundamental Theorem of Calculus. The Mean Value Theorem for integrals. Methods for integrations: integration by parts, change of variables in an integral.

Evaluation system and criteria

The exam consists of a written test and four e-tivities carried out during the course. The written test normally includes 6 numerical exercises on the main topics covered in the course (3 on the Subjects 1 and 2, 3 on the

Subjects 3 and 4). It is possible to split the exam in 2 parts. During the written test, it is NOT allowed to use handouts, notes, texts or forms. E-tivities are evaluated from 0 to 4 points, while 0-30 points are assigned at the written test.

Bibliography and resources

4. Materials to consult

The educational materials (lectures notes, slides and video lessons) are available on the Unicusano platform.

5. Recommended bibliography

Suggested readings are:

Robert A. Adams, Christopher Essex. Calculus: A Complete Course, 9th Edition. Pearson 2018

Frank Ayres and Elliott Mendelson. Schaum's Outline of Calculus, 6th Edition. McGraw Hill 2013

Elliott Mendelson. Schaum's 3000 Solved Problems in Calculus. McGraw Hill Education 2009