UNICUSANO
ROME

Code: MATH-02/B Geometry **Credits: 6**
Matter: Advanced geometry
Main language of instruction: Italian
Other language of instruction: English

Teaching Staff
Head instructor:
Prof. Alfredo Donno - alfredo.donno@unicusano.it

Objectives

It is required that the student attending this course has a good knowledge of the program of the courses of Geometry, Calculus 1 and Calculus 2.
The aim of the course is to apply the methods of Mathematical analysis to the study of the local properties of curves and surfaces of the space, also by using techniques coming from Linear algebra and matrix calculus. The main goal is to understand fundamental notions, in particular the curvature and torsion of a curve, by using the Frenet frame and the Frenet formulas, as well as the importance of the classifications of the points of a surface into elliptic, parabolic and hyperbolic points.

Competencies:

1. *Knowledge and understanding:* At the end of the course, it is expected that the student has a good knowledge of the tensor and differential calculus methods. The student should know the basic definitions and properties of curves and surfaces in the space, both in their Cartesian and parametric representation.
2. *Applying knowledge and understanding:* At the end of the course, the student should be able to apply the analytic techniques to the investigation of the local properties of parametric curves and parametric surfaces of the space: determination and interpretation of the Frenet frame at a point of a curve, as well as of the Gaussian curvature at a point of a surface.

Syllabus

Program of the course:

Subject 1 (Modules 1 – 2): BASIC VECTOR CALCULUS
Operations on geometric vectors: sum, scalar multiplication, scalar product, vector product, mixed product. Multivariable calculus. Differentiation of multivariable functions, of scalar products and of vector products.

Subject 2 (Modules 3 – 6): BASIC TENSOR ALGEBRA

Real vector spaces: definitions and examples. Linear dependence and independence. Vector subspaces, generators, bases and coordinates. Euclidean vector spaces. Tensors. Symmetric and skewsymmetric tensors. Tensor product of two vectors. The Euclidean vector space of tensors and its properties. Rotations. Skewsymmetric tensors and vector products. Eigenvalues and eigenvectors. Spectral theorem. Polar decomposition Theorem. Cayley-Hamilton Theorem.

Subject 3 (Modules 7 – 8): REAL QUADRATIC FORMS

Real quadratic forms: definitions. Real quadratic forms and symmetric matrices. Matrices associated with quadratic forms and symmetric operators on $\mathbf{R}^n$. Nullity and signature of a quadratic form. Congruence invariants. The standard form of a quadratic real form.

Subject 4 (Modules 8 – 10): CURVES AND SURFACES IN THE SPACE

Cartesian equation of a surface. Analytic representation of curves in the space. The sphere. The circle in the space. Cones. Cylinders. Surfaces of revolution.

Subject 5 (Modules 11 – 14): DIFFERENTIAL GEOMETRY OF CURVES

Parametric curves: first definitions. Regular curves: the tangent line and the osculator plane. Length of an arc of curve. Natural parametrization. The Frenet moving frame. Curvature, torsion and Frenet formulas. Rigidity theorem. Osculator circle. Circular helix. Evolutes and evolvents.

Subject 6 (Modules 14 – 18): DIFFERENTIAL GEOMETRY OF SURFACES

Parametric representation of a surface: first definitions. Regular surfaces. The first and the second parameter curve. Tangent plane and unit normal vector. The first and the second fundamental quadratic form. Meusnier Theorem. Curvature of a curve on a surface. Normal curvature, principal curvatures and principal directions. Euler Theorem. Gaussian curvature and mean curvature. Classification of the points of a regular surface. Osculating quadric. Ruled surfaces: cones, cylinders, tangent surfaces. Developable ruled surfaces.

Evaluation system and criteria

Written examination.
Maximum mark: 30/30 cum Laude.
Details:
25/30: Four exercises;
03/30: E-tivity 1: Circular helix;
03/30: E-tivity 2: The tangent surface as an example of a ruled surface.

Bibliography and resources

1. Materials to consult:

Teaching materials are provided by the teacher. The educational material on the platform is divided into 18 modules. They completely cover the program and each of them contains lecture notes, slides and video lessons in which the teacher comments on the slides. This material contains all the elements necessary to deal with the study of the course.

2. Recommended bibliography:

- Serge Lang, Linear Algebra. Undergraduate texts in Mathematics, 1987. Springer.
- M. Barot, J.A. Jiménez Gonzalez, J.A. de la Pena, Quadratic forms: combinatorics and numerical results, Algebra and Applications, Springer, 2019.
- A. Pressley, Elementary differential geometry, 2nd edition, Springer, 2010.
- M. Abate, F. Tovena, Curves and surfaces, Springer-Verlag Milan 2012.